

This tutorial is prepared by Lingyan Gong of Wenzhou University. It is aimed to help the readers to build the ICP-based preconcentration model using Comsol v5.2 software. Backgrounds, theories, methods, and key results are described in the following paper:

Zirui Li, Wei Liu, Lingyan Gong, Yudan Zhu, Yuantong Gu, Jongyoon Han, "Accurate multi physics numerical analysis of particle preconcentration based on ion concentration polarization", *International Journal of Applied Mechanics*, **9**(8), 1750107, 2017. <http://dx.doi.org/10.1142/S1758825117501071>

### *Modeling Instructions*

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From the **File** menu, choose **New**.

#### **NEW**

In the **New** window, click **Model Wizard**.

#### **MODEL WIZARD**

- 1 In the **Model Wizard** window, click **2D**.
- 2 In the **Select Physics** tree, select **Fluid Flow>Single-Phase Flow>Creeping Flow (spf)**.
- 3 Click **Add**.
- 4 In the **Select Physics** tree, select **AC/DC>Electrostatics(es)**.
- 5 Click **Add**.
- 6 In the **Select Physics** tree, select **Chemical Species Transport>Transport of Dilutes Species (tds)**.
- 7 Click **Add**.
- 8 In the **Concentration** table, enter the following setting:  

c1
----
- 9 In the **Select Physics** tree, select **Chemical Species Transport>Transport of Dilutes Species (tds)**.
- 10 Click **Add**.
- 11 In the **Concentration** table, enter the following setting:  

c2
----
- 12 In the **Select Physics** tree, select **Chemical Species Transport>Transport of Dilutes Species (tds)**.

**13** Click **Add**.

**14** In the **Concentration** table, enter the following setting:

$$\frac{c_3}{c_0}$$

**15** Click **Study**.

**16** In the **Select Study** tree, select **Preset Studies for Selected Physics Interfaces>Stationary**.

**17** Click **Done**.

## GLOBAL DEFINITIONS

### Parameters

- 1 On the **Home** toolbar, click **Parameters**.
- 2 In the **Settings** window for Parameters, locate the **Parameters** section.
- 3 In the table, enter the following setting:

Name	Expression	Value	Description
W	120[um]	1.2E-4 m	Length of microchannel
H	2[um]	2E-6 m	Width of microchannel
sigma	-5[mC/m <sup>2</sup> ]	-0.005 C/m <sup>2</sup>	Surface charge density
phi0	25.8[mV]	0.0258 V	Thermal voltage
VL	20	20	Voltage at inlet (in thermal voltage)
VR	0	0	Voltage at outlet (in thermal voltage)
vcm	0	0	Cross-membrane Voltage (in thermal voltage)
T	300[K]	300 K	Temperature
rho	1e3[kg/m <sup>3</sup> ]	1000 kg/m <sup>3</sup>	Mass density of water
eta	1e-3[Pa*s]	0.001 Pa*s	Dynamic Viscosity
c0	1[mM]	1 mol/m <sup>3</sup>	Concentration of K+
D1	1.97*1e-9[m <sup>2</sup> /s]	1.97E-9 m <sup>2</sup> /s	Diffusion coefficient of K+
D2	2.03*1e-9[m <sup>2</sup> /s]	2.03E-9 m <sup>2</sup> /s	Diffusion coefficient of Cl-
D3	0.51*1e-9[m <sup>2</sup> /s]	5.10E-10 m <sup>2</sup> /s	Diffusion coefficient of P2-
rc0	1e-7	1E-7	Ratio of particle concentration in the reservoir to that of Cl-

## DEFINITIONS

Create an random function for the simulation of initial concentrations of ions and particles.

- 1 In the **Home** toolbar, click **Functions>Local>Random**.

- 2 In the **Function name** text field, type **ran**.
- 3 In the **Number of arguments** text field, type **2**.
- 4 From the **Distribution** list, choose **Normal**.

## **GEOMETRY 1**

- 1 In the **Model Builder** window, under **Component 1 (comp1)** click **Geometry 1**.
- 2 In the **Settings** window for Geometry, locate the **Units** section.
- 3 From the **Length unit** list, choose **µm**.
- 4 Locate the **Advanced** section. From the **Default repair tolerance** list, choose **Relative**.

### *Rectangle 1 (r1)*

- 1 On the **Geometry** toolbar, click **Primitives** and choose **Rectangle**.
- 2 In the **Settings** window for Rectangle, locate the **Size and Shape** section.
- 3 In the **Width** text field, type **240**.
- 4 In the **Height** text field, type **H**.
- 5 Locate the **Position** section. In the **y** text field, type **-2**.

### *Rectangle 2 (r2)*

- 1 On the **Geometry** toolbar, click **Primitives** and choose **Rectangle**.
- 2 In the **Settings** window for Rectangle, locate the **Size and Shape** section.
- 3 In the **Width** text field, type **60**.
- 4 In the **Height** text field, type **60**.
- 5 Locate the **Position** section. In the **y** text field, type **-60**.

### *Rectangle 3 (r3)*

- 6 On the **Geometry** toolbar, click **Primitives** and choose **Rectangle**.
- 7 In the **Settings** window for Rectangle, locate the **Size and Shape** section.
- 8 In the **Width** text field, type **60**.
- 9 In the **Height** text field, type **60**.
- 10 Locate the **Position** section. In the **x** text field, type **180**.
- 11 Locate the **Position** section. In the **y** text field, type **-60**.

### *Point 1 (pt1)*

- 1 On the **Geometry** toolbar, click **Primitives** and choose **Point**.
- 2 In the **Settings** window for Point, locate the **Point** section.
- 3 In the **x** text field, type **119**.

### *Point 2 (pt2)*

- 1 On the **Geometry** toolbar, click **Primitives** and choose **Point**.

- 2 In the **Settings** window for Point, locate the **Point** section.
- 3 In the **x** text field, type 121.

*Point 3 (pt3)*

- 1 On the **Geometry** toolbar, click **Primitives** and choose **Point**.
- 2 In the **Settings** window for Point, locate the **Point** section.
- 3 In the **x** text field, type 119.
- 4 In the **y** text field, type -H.

*Point 4 (pt4)*

- 1 On the **Geometry** toolbar, click **Primitives** and choose **Point**.
- 2 In the **Settings** window for Point, locate the **Point** section.
- 3 In the **x** text field, type 121.
- 4 In the **y** text field, type -H.

*Form Union (fin)*

In the **Model Builder** window, under **Component 1 (comp1)>Geometry 1** right-click **Form Union (fin)** and choose **Build Selected**.

**CREEPING FLOW (SPF)**

- 1 In the **Model Builder** window, under **Component 1 (comp1)** click **Creeping Flow (spf)**.
- 2 In the **Model Builder** window's toolbar, click the **Show** button and select **Stabilization** in the menu.
- 3 In the **Settings** window for Creeping Flow, type Creeping in the **Label** text field.
- 4 Click to expand the **Consistent Stabilization** section, then clear the **Crosswind diffusion** check box.

**CREEPING (SPF)**

*Transport Properties 1*

- 1 In the **Model Builder** window, expand the **Component 1 (comp1)>Creeping (spf)** node, then click **Fluid Properties 1**.
- 2 In the **Settings** window for Fluid properties, locate the **Fluid Properties** section.
- 3 From the  $\rho$  list, choose **User defined**. In the associated text field, type rho.
- 4 From the  $\mu$  list, choose **User defined**. In the associated text field, type eta.

*Volume Force 1*

- 1 On the **Physics** toolbar, click **Domains** and choose **Volume Force**.
- 2 In the **Settings** window for Volume Force, locate the **Domain Selection** section. From the **Selection** list choose **All Domains**.
- 3 Locate the **Volume Force** section. in the **Volume force** table, enter the following setting:

$(c1-c2-2*c3)*(es.Ex)*e\_const*N\_A\_const$	x
---	---

$(c1-c2-2*c3)*(es.Ey)*e\_const*N\_A\_const$	y
---	---

#### *Symmetry 1*

- 1 On the **Physics** toolbar, click **Boundaries** and choose **Symmetry**.
- 2 Select boundaries 5, 9, 11, 13 and 18 only.
- 3 In the **Settings** window for Symmetry, locate the **Boundary Selection** section. Click the **Create Selection** button and type Center Line in the pop-up window. Click **OK**, then **Center Line** is shown under the **Definitions** node in the **Model Builder** window.

#### *Inlet 1*

- 1 On the **Physics** toolbar, click **Boundaries** and choose **Inlet**.
- 2 Select boundaries 1 and 3 only.
- 3 In the **Settings** window for Inlet, locate the **Boundary Condition** section. From the list, choose **Pressure**.

#### *Outlet 1*

- 1 On the **Physics** toolbar, click **Boundaries** and choose **Outlet**.
- 2 Select boundaries 19 and 20 only.
- 3 In the **Settings** window for Outlet, locate the **Pressure Condition** section, select the **Normal flow** check box.

#### *Pressure Point Constraint 1*

- 1 On the **Physics** toolbar, click **Points** and choose **Pressure Point Constraint**.
- 2 Select point 14 only.

### **ELECTROSTATICS (ES)**

- 1 In the **Model Builder** window, under **Component 1 (comp1)** click **Electrostatics (es)**.
- 2 In the **Settings** window for Electrostatics, locate **Thickness** section. In the **Out-of-plane thickness** text field, type 10[um].

#### *Charge Conservation 1*

- 1 In the **Model Builder** window, expand the **Component 1 (comp1) > Electrostatics (es)** node, then click **Charge Conservation 1**.
- 2 In the **Settings** window for Charge Conservation, locate the **Electric Field** section.
- 3 From the **Relative permittivity** list, choose **User defined**. In the associated text field, type 78.

#### *Space Charge Density 1*

- 1 On the **Physics** toolbar, click **Domains** and choose **Space Charge Density**.
- 2 In the **Settings** window for Space Charge Density, locate the **Domain Selection** section. From the Selection list, choose **All domains**.
- 3 Locate the **Space Charge Density** section. From the **Space charge density** list, choose **User defined**. In the associated text field, type  $(c1-c2-2*c3)*e\_const*N\_A\_const$ .

### *Surface Charge Density 1*

- 1 On the **Physics** toolbar, click **Boundaries** and choose **Surface Charge Density**.
- 2 Select boundaries 8 and 12 only.
- 3 In the **Settings** window for Surface Charge Density, locate the **Surface Charge Density** section. In the **Surface charge density** text field, type sigma.

### *Electric Potential 1*

- 1 On the **Physics** toolbar, click **Boundaries** and choose **Electric Potential**.
- 2 Select boundaries 1 and 3 only.
- 3 In the **Settings** window for Electric Potential, locate the **Electric Potential** section. In the **Electric potential** text field, type  $V_L \cdot \phi_0$ .

### *Electric Potential 2*

- 1 On the **Physics** toolbar, click **Boundaries** and choose **Electric Potential**.
- 2 Select boundaries 19 and 20 only.
- 3 In the **Settings** window for Electric Potential, locate the **Electric Potential** section. In the **Electric potential** text field, type  $V_R \cdot \phi_0$ .

### *Electric Potential 3*

- 1 On the **Physics** toolbar, click **Boundaries** and choose **Electric Potential**.
- 2 Select boundary 10 only.
- 3 In the **Settings** window for Electric Potential, locate the **Electric Potential** section. In the **Electric potential** text field, type  $((V_L + V_R)/2 - v_{cm}) \cdot \phi_0$ .

## **TRANSPORT OF DILUTED SPECIES (TDS)**

- 1 In the **Model Builder** window, under **Component 1 (comp1)** click **Transport of Diluted Species (tds)**.
- 2 In the **Settings** window for Transport of Diluted Species, type c1 Species in the **Label** text field.
- 3 Locate the **Transport Mechanism** section. Select the **Migration in electric field** check box.
- 4 Click to expand the **Consistent Stabilization** section, then clear the **Crosswind diffusion** check box.
- 5 Click to expand the **Advanced Settings** section. From the **Convective term** list, choose **Conservative form**.
- 6 Click to expand the **Discretization** section. From the **Concentration** list, choose **Quadratic**.

## **C1 SPECIES (TDS)**

### *Transport Properties 1*

- 1 In the **Model Builder** window, expand the **Component 1 (comp1)**>**c1 Species (tds)** node, then click **Transport Properties 1**.

- 2 In the **Settings** window for Transport Properties, locate the **Model Inputs** section. From the **Velocity field** list, choose **Velocity field (spf)**. From the **Electric potential** list, choose **Electric potential (es)**. In the **Temperature** text field, type T.
- 3 Locate the **Diffusion** section. In the **Diffusion coefficient** text field, type D1.
- 4 Locate the **Migration in Electric Field** section. In the **Charge number** text field, type 1.

#### *Initial Values 1*

- 1 In the **Model Builder** window, click **Component 1 (comp1)>c1 Species (tds)>Initial Values 1**.
- 2 In the **Settings** window for Initial Values, locate the **Initial Values** section. In the **Concentration** text field, type  $c0*(1+0.0001*ran(x[1/m],y[1/m]))$ .

#### *Inflow 1*

- 1 On the **Physics** toolbar, click **Boundaries** and choose **Inflow**.
- 2 Select boundaries 1 and 3 only.
- 3 In the **Settings** window for Inflow, locate the **Concentration** section. Type c0 in the text field.

#### *Outflow 1*

- 1 On the **Physics** toolbar, click **Boundaries** and choose **Outflow**.
- 2 Select boundaries 19 and 20 only.

#### *Symmetry 1*

- 1 On the **Physics** toolbar, click **Boundaries** and choose **Symmetry**.
- 2 In the **Settings** window for Symmetry, locate the **Boundary Selection** section. From the **Selection list**, choose **Center Line**.

#### *Concentration 1*

- 1 On the **Physics** toolbar, click **Boundaries** and choose **Concentration**.
- 2 Select boundaries 10 only.
- 3 In the **Settings** window for Concentration, locate the **Concentration** section, select the **Species c1** check box and type  $c0*2$  in the associated text field.

### **TRANSPORT OF DILUTED SPECIES 2 (TDS2)**

- 1 In the **Model Builder** window, under **Component 1 (comp1)** click **Transport of Diluted Species 2 (tds2)**.
- 2 In the **Settings** window for Transport of Diluted Species, type c2 Species in the **Label** text field.
- 3 Locate the **Transport Mechanism** section. Select the **Migration in electric field** check box.
- 4 Click to expand the **Consistent Stabilization** section, then clear the **Crosswind diffusion** check box.
- 5 Click to expand the **Advanced Settings** section. From the **Convective term** list, choose **Conservative form**.
- 6 Click to expand the **Discretization** section. From the **Concentration** list, choose **Quadratic**.

## C2 SPECIES (TDS2)

### *Transport Properties 1*

- 1 In the **Model Builder** window, expand the **Component 1 (comp1)>c2 Species (tds2)** node, then click **Transport Properties 1**.
- 2 In the **Settings** window for Transport Properties, locate the **Model Inputs** section. From the **Velocity field** list, choose **Velocity field (spf)**. From the **Electric potential** list, choose **Electric potential (es)**. In the **Temperature** text field, type T.
- 3 Locate the **Diffusion** section. In the **Diffusion coefficient** text field, type D2.
- 4 Locate the **Migration in Electric Field** section. In the **Charge number** text field, type -1.

### *Initial Values 1*

- 1 In the **Model Builder** window, click **Component 1 (comp1)>c2 Species (tds2)>Initial Values 1**.
- 2 In the **Settings** window for Initial Values, locate the **Initial Values** section. In the **Concentration** text field, type  $c0*(1-2*rc0)$ .

### *Inflow 1*

- 1 On the **Physics** toolbar, click **Boundaries** and choose **Inflow**.
- 2 Select boundaries 1 and 3 only.
- 3 In the **Settings** window for Inflow, locate the **Concentration** section. Type  $c0*(1-2*rc0)$  in the text field.

### *Outflow 1*

- 1 On the **Physics** toolbar, click **Boundaries** and choose **Outflow**.
- 2 Select boundaries 19 and 20 only.

### *Symmetry 1*

- 1 On the **Physics** toolbar, click **Boundaries** and choose **Symmetry**.
- 2 In the **Settings** window for Symmetry, locate the **Boundary Selection** section. From the **Selection list**, choose **Center Line**.

## TRANSPORT OF DILUTED SPECIES 3 (TDS3)

- 1 In the **Model Builder** window, under **Component 1 (comp1)** click **Transport of Diluted Species 3 (tds3)**.
- 2 In the **Settings** window for Transport of Diluted Species, type c3 Species in the **Label** text field.
- 3 Locate the **Transport Mechanism** section. Select the **Migration in electric field** check box.
- 4 Click to expand the **Consistent Stabilization** section, then clear the **Crosswind diffusion** check box.
- 5 Click to expand the **Advanced Settings** section. From the **Convective term** list, choose **Conservative form**.
- 6 Click to expand the **Discretization** section. From the **Concentration** list, choose **Quadratic**.



### C 3 SPECIES (TDS3)

#### *Transport Properties 1*

- 1 In the **Model Builder** window, expand the **Component 1 (comp1)>c3 Species (tds3)** node, then click **Transport Properties 1**.
- 2 In the **Settings** window for Transport Properties, locate the **Model Input** section. From the **Velocity field** list, choose **Velocity field (spf)**. From the **Electric potential** list, choose **Electric potential (es)**. In the **Temperature** text field, type T.
- 3 Locate the **Diffusion** section. In the **Diffusion coefficient** text field, type D3.
- 4 Locate the **Migration in Electric Field** section. In the **Charge number** text field, type -2.

#### *Initial Values 1*

- 1 In the **Model Builder** window, click **Component 1 (comp1)>c3 Species (tds3)>Initial Values 1**.
- 2 In the **Settings** window for Initial Values, locate the **Initial Values** section. In the **Concentration** text field, type  $c0 * rc0$ .

#### *Inflow 1*

- 1 On the **Physics** toolbar, click **Boundaries** and choose **Inflow**.
- 2 Select boundaries 1 and 3 only.
- 3 In the **Settings** window for Inflow, locate the **Concentration** section. Type  $c0 * rc0$  in the text field.

#### *Outflow 1*

- 1 On the **Physics** toolbar, click **Boundaries** and choose **Outflow**.
- 2 Select boundaries 19 and 20 only.

#### *Symmetry 1*

- 1 On the **Physics** toolbar, click **Boundaries** and choose **Symmetry**.
- 2 In the **Settings** window for Symmetry, locate the **Boundary Selection** section. From the **Selection list**, choose **Center Line**.

### MESH 1

In the **Model Builder** window, under **Component 1 (comp1)** right-click **Mesh 1** and choose **More Operations>Edge**.

#### *Edge 1*

- 1 In the **Settings** window for Edge, locate the **Boundary Selection** section. Click the **Paste Selection** button, and type 2, 4-5, 8-13, 15, 17-18 in the text field of the pop-up window.
- 2 In the **Model Builder** window, under **Component 1 (comp1)>Mesh 1** right-click **Edge 1** and choose **Distribution**.

#### *Distribution 1*

- 1 Select Boundaries 8, 9, 12 and 13 only.
- 2 In the **Settings** window for Distribution, locate the **Distribution** section.

- 3 From the **Distribution properties** list, choose **Predefined distribution type**.
- 4 In the **Number of elements** text field, type 300.
- 5 In the **Element ratio** text field, type 300.
- 6 From the **Distribution method**, choose the **Geometric sequence**.
- 7 Select the **Symmetric distribution** check box.

#### *Edge 1*

In the **Model Builder** window, under **Component 1 (comp1)>Mesh 1** right-click **Edge 1** and choose **Distribution**.

#### *Distribution 2*

- 1 Select Boundaries 10 and 11 only.
- 2 In the **Settings** window for Distribution, locate the **Distribution** section.
- 3 From the **Distribution properties** list, choose **Predefined distribution type**.
- 4 In the **Number of elements** text field, type 60.
- 5 In the **Element ratio** text field, type 30.
- 6 From the **Distribution method**, choose the **Geometric sequence**.
- 7 Select the **Symmetric distribution** check box.

#### *Edge 1*

In the **Model Builder** window, under **Component 1 (comp1)>Mesh 1** right-click **Edge 1** and choose **Distribution**.

#### *Distribution 3*

- 1 Select Boundaries 15, 17 and 18 only.
- 2 In the **Settings** window for Distribution, locate the **Distribution** section.
- 3 From the **Distribution properties** list, choose **Predefined distribution type**.
- 4 In the **Number of elements** text field, type 80.
- 5 In the **Element ratio** text field, type 2000.
- 6 From the **Distribution method**, choose the **Geometric sequence**.
- 7 Select the **Reverse distribution** check box.

#### *Edge 1*

In the **Model Builder** window, under **Component 1 (comp1)>Mesh 1** right-click **Edge 1** and choose **Distribution**.

#### *Distribution 4*

- 1 Select Boundaries 2, 4 and 5 only.
- 2 In the **Settings** window for Distribution, locate the **Distribution** section.
- 3 From the **Distribution properties** list, choose **Predefined distribution type**.
- 4 In the **Number of elements** text field, type 80.

- 5 In the **Element ratio** text field, type 2000.
- 6 From the **Distribution method**, choose the **Geometric sequence**.

#### *Mapped 1*

- 1 In the **Model Builder** window, under **Component 1 (comp1)** right-click **Mesh 1** and choose **Mapped**.
- 2 In the **Settings** window for Mapped, Locate the **Domain Selection** section. From the **Geometric entity level** list, choose **Domain**.
- 3 Select domains 2, 3 and 5 only.
- 4 In the **Model Builder** window, under **Component 1 (comp1)>Mesh 1** right-click **Mapped 1** and choose **Distribution**.

#### *Distribution 1*

- 1 Select boundaries 3, 7, 16 and 20 only.
- 2 In the **Settings** window for Distribution, locate the **Distribution** section.
- 3 From the **Distribution properties** list, choose **Predefined distribution type**.
- 4 In the **Number of elements** text field, type 40.
- 5 In the **Element ratio** text field, type 12000.
- 6 From the **Distribution method**, choose the **Geometric sequence**.

#### *Mapped 2*

- 1 In the **Model Builder** window, under **Component 1 (comp1)** right-click **Mesh 1** and choose **Mapped**.
- 2 In the **Settings** window for Mapped, Locate the **Domain Selection** section. From the **Geometric entity level** list, choose **Domain**.
- 3 Select domains 1 and 4 only.
- 4 In the **Model Builder** window, under **Component 1 (comp1)>Mesh 1** right-click **Mapped 2** and choose **Distribution**.

#### *Distribution 1*

- 1 Select boundaries 1, 6, 14 and 19 only.
- 2 In the **Settings** window for Distribution, locate the **Distribution** section.
- 3 From the **Distribution properties** list, choose **Predefined distribution type**.
- 4 In the **Number of elements** text field, type 40.
- 5 In the **Element ratio** text field, type 1200000.
- 6 From the **Distribution method**, choose the **Geometric sequence**.
- 7 Select the **Reverse distribution** check box.
- 8 Click **Build All**.

## STUDY 1

- 1 In the **Model Builder** window, under **Component 1 (comp1)** click **Study 1**.
- 2 In the **Settings** window for study, type Study 1 (reference initial) in the **Label** text field.
- 3 Locate the **Study Setting** section, clear the **Generate default plots** check box.

### *Step 1: Stationary*

- 1 In the **Model Builder** window, under **Study 1** click **Step 1: Stationary**.
- 2 In the **Settings** window for Stationary, click to expand the **Study extensions** section.
- 3 Locate the **Study Extensions** section. Select the **Auxiliary sweep** check box.
- 4 Click the **Add**.
- 5 In the table, enter the following settings:

Parameter name	Parameter value list	Parameter unit
vcm (Cross-membrane Voltage (in thermal voltage))	2	

- 6 On the **Study** toolbar, click **Compute**.

## ADD STUDY

- 1 On the **Home** toolbar, click **Add Study** to open the **Add Study** window.
- 2 Go to the **Add Study** window.
- 3 Find the **Studies** subsection. In the **Select Study** tree, select **Preset Studies>Stationary**.
- 4 Click **Add Study** in the window toolbar.
- 5 On the **Home** toolbar, click **Add Study** again to close the Add Study window.

## STUDY 2

- 1 In the **Model Builder** window, under **Component 1 (comp1)** click **Study 2**.
- 2 In the **Settings** window for study, type Study 2 (changing vcm) in the **Label** text field.
- 3 Locate the **Study Setting** section, clear the **Generate default plots** check box.

### *Step 1: Stationary*

1. In the **Model Builder** window, under **Study 1** click **Step 1: Stationary**.
2. In the **Settings** window for Stationary, locate the **Values of Dependent Variables** section.
3. Find the **Initial values of variables solved for** subsection. From the **Settings** list, choose **User controlled**.
4. From the **Method** list, choose **Solution**.
5. From the **Study** list, choose **Study 1 (reference initial), Stationary**.

6. From the **Parameter value (vcm)** list, choose **2**.
7. Find the **Values of variables not solved for** subsection. From the **Settings** list, choose **User controlled**.
8. From the **Method** list, choose **Solution**.
9. From the **Study** list, choose **Study 1 (reference initial), Stationary**.
10. From the **Parameter value (vcm)** list, choose **2**.
11. Locate the **Study Extensions** section.
12. Select the **Auxiliary sweep** check box.
13. Click the **Add**.
14. In the table, enter the following settings:

Parameter name	Parameter value list	Parameter unit
vcm (Cross-membrane Voltage(in thermal voltage))	range(2,1,50)	

15. From the **Run continuation for** list, choose **No parameter**.
16. From the **Reuse solution from previous step** list, choose **Yes**.
17. On the **Study** toolbar, click **Compute**.

## RESULTS

### *1D Plot Group 1*

1. On the **Home** toolbar, click **Add Plot Group** and choose **1D Plot Group**.
2. In the **Settings** window for 1D Plot Group, type Enrichment factor in the **Label** text field.
3. Locate the **Data** section. From the **Data set** list, choose **Study 2 (changing vcm)/Solution 2(sol2)**.
4. From the **Parameter selection (vcm)** list, choose **From list**.
5. From the **Parameter values (vcm)** list, choose **4, 10, 14, 17, 20, 23, 26, 34** and **50**.
6. Locate the **Plot Settings** section. Select the **x-axis label** check box and type x( $\mu\text{m}$ ) in the associated text field.
7. Locate the **Plot Settings** section. Select the **y-axis label** check box and type Enrichment factor in the associated text field.
8. Locate the **Axis** section, select the **y-axis log scale** check box.
9. Click to expand the **Legend** section. Select the **Show legends** check box.

### *Line Graph 1*

1. On the **Enrichment factor** toolbar, click **Line Graph**.
2. In the **Settings** window for Line Graph, locate the **Selection** section.
3. From the **Selection** list, choose **Center Line**.

4. Locate the **y-Axis Data** section. In the **Expression** text field, type  $c3/(c0*rc0)$ .
5. Click to expand the **Coloring and Style** section. In the **Width** text field, type 2.
6. Click to expand the **Legends** section. Select the **Show legends** check box.
7. From the **Legends** list, choose **Manual**.
8. In the **Legends** table, enter the following setting:

<b>Legends</b>
vcm=4 VT
vcm=10 VT
vcm=14 VT
vcm=17 VT
vcm=20 VT
vcm=23 VT
vcm=26 VT
vcm=34 VT
vcm=50 VT

9. On the **Enrichment factor** toolbar, click **Plot**.

#### *1D Plot Group 2*

- 1 On the **Home** toolbar, click **Add Plot Group** and choose **1D Plot Group**.
- 2 In the **Settings** window for 1D Plot Group, type Concentration (a) in the **Label** text field.
- 3 Locate the **Data** section. From the **Data set** list, choose **Study 2 (changing vcm)/Solution 2(sol2)**.
- 4 From the **Parameter selection (vcm)** list, choose **From list**.
- 5 From the **Parameter values (vcm)** list, choose **14**.
- 6 Locate the **Plot Settings** section. Select the **x-axis label** check box and type  $x(\mu\text{m})$  in the associated text field.
- 7 Locate the **Axis** section, select the **y-axis log scale** check box.
- 8 Locate the **Legend** section, select the **Show legends** check box.

#### *Line Graph 1*

- 1 On the **Concentration (a)** toolbar, click **Line Graph**.
- 2 In the **Settings** window for Line Graph, type c1 in the **Label** text field.
- 3 Locate the **Selection** section. From the **Selection** list, choose **Center Line**.
- 4 Locate the **y-Axis Data** section. Click **Replace Expression** in the upper-right corner of the **y-axis data** section. From the menu, choose **Model>Component 1>c1 Species>c1 - Concentration**.
- 5 Click to expand the **Coloring and Style** section. In the **Width** text field, type 2.
- 6 Click to expand the **Legends** section. Select the **Show legends** check box.
- 7 From the **Legends** list, choose **Manual**.

8 In the **Legends** table, enter the following setting:

<b>Legends</b>
K+

#### *Line Graph 2*

- 1 On the **Concentration (a)** toolbar, click **Line Graph**.
- 2 In the **Settings** window for Line Graph, type c2 in the **Label** text field.
- 3 Locate the **Selection** section. From the **Selection** list, choose **Center Line**.
- 4 Locate the **y-Axis Data** section. Click **Replace Expression** in the upper-right corner of the **y-axis data** section. From the menu, choose **Model>Component 1>c2 Species>c2 - Concentration**.
- 5 Click to expand the **Coloring and Style** section.
- 6 From the **Line** list, choose **Dashed**.
- 7 In the **Width** text field, type 2.
- 8 Click to expand the **Legends** section. Select the **Show legends** check box.
- 9 From the **Legends** list, choose **Manual**.
- 10 In the **Legends** table, enter the following setting:

<b>Legends</b>
Cl-

#### *Line Graph 3*

- 1 On the **Concentration (a)** toolbar, click **Line Graph**.
- 2 In the **Settings** window for Line Graph, type c3 in the **Label** text field.
- 3 Locate the **Selection** section. From the **Selection** list, choose **Center line**.
- 4 Locate the **y-Axis Data** section. Click **Replace Expression** in the upper-right corner of the **y-axis data** section. From the menu, choose **Model>Component 1>c3 Species>c3 - Concentration**.
- 5 Click to expand the **Coloring and Style** section. In the **Width** text field, type 2.
- 6 Click to expand the **Legends** section. Select the **Show legends** check box.
- 7 From the **Legends** list, choose **Manual**.
- 8 In the **Legends** table, enter the following setting:

<b>Legends</b>
P2-

- 9 On the **Concentration (a)** toolbar, click **Plot**.

#### *1D Plot Group 3*

- 1 In the **Model Builder** window, right click **Results>Concentration (a)**. Choose **Duplicate**.
- 2 In the **Settings** window for 1 D Plot Group, type Concentration (b) in the **Label** text field.

- 3 Locate the **Data** section. From the **Parameter values (vcm)** list, choose **34**.
- 4 On the **Concentration (b)** toolbar, click **Plot**.

#### *1D Plot Group 4*

- 1 On the **Home** toolbar, click **Add Plot Group** and choose **1D Plot Group**.
- 2 In the **Settings** window for 1D Plot Group, type Potential in the **Label** text field.
- 3 Locate the **Data** section. From the **Data set** list, choose **Study 2 (changing vcm)/Solution 2(sol2)**.
- 4 From the **Parameter selection (vcm)** list, choose **From list**.
- 5 From the **Parameter values (vcm)** list, choose **4, 14** and **34**.
- 6 Locate the **Plot Settings** section. Select the **x-axis label** check box and type  $x(\mu\text{m})$  in the associated text field.
- 7 Select the **y-axis label** check box and type  $\Phi/V_T$  in the associated text field.
- 8 Locate the **Legend** section, select the **Show legends** check box.

#### *Line Graph 1*

- 1 On the **Potential** toolbar, click **Line Graph**.
- 2 Locate the **Selection** section. From the **Selection** list, choose **Center line**.
- 3 Locate the **y-Axis Data** section. In the **Expression** text field, type  $V/\phi_0$ .
- 4 Click to expand the **Coloring and Style** section. In the **Width** text field, type 2.
- 5 Click to expand the **Legends** section. Select the **Show legends** check box.
- 6 From the **Legends** list, choose **Manual**.
- 7 In the **Legends** table, enter the following setting:

<b>Legends</b>
vcm=4 VT
vcm=14 VT
vcm=34 VT

- 8 On the **Potential** toolbar, click **Plot**.

#### *1D Plot Group 5*

- 1 On the **Home** toolbar, click **Add Plot Group** and choose **1D Plot Group**.
- 2 In the **Settings** window for 1D Plot Group, type Ex in the **Label** text field.
- 3 Locate the **Data** section. From the **Data set** list, choose **Study 2 (changing vcm)/Solution 2(sol2)**.
- 4 From the **Parameter selection (vcm)** list, choose **From list**.
- 5 From the **Parameter values (vcm)** list, choose **4, 14** and **34**.
- 6 Locate the **Plot Settings** section. Select the **x-axis label** check box and type  $x(\mu\text{m})$  in the associated text field.
- 7 Select the **y-axis label** check box and type  $E_x (V/m)$  in the associated text field.



- 8 Locate the **Legend** section, select the **Show legends** check box.

#### *Line Graph 1*

- 1 On the **Ex** toolbar, click **Line Graph**.
- 2 Locate the **Selection** section. From the **Selection** list, choose **Center line**.
- 3 Locate the **y-Axis Data** section. Click **Replace Expression** in the upper-right corner of the **y-axis data** section. From the menu, choose **Model>Component 1>Electrostatics>Electric> Electric field>es.Ex – Electric field, x component**.
- 4 Click to expand the **Coloring and Style** section. In the **Width** text field, type 2.
- 5 Click to expand the **Legends** section. Select the **Show legends** check box.
- 6 From the **Legends** list, choose **Manual**.
- 7 In the **Legends** table, enter the following setting:

<b>Legends</b>
vcm=4 VT
vcm=14 VT
vcm=34 VT

- 8 On the **Ex** toolbar, click **Plot**.

#### *1D Plot Group 6*

- 1 On the **Home** toolbar, click **Add Plot Group** and choose **1D Plot Group**.
- 2 In the **Settings** window for 1D Plot Group, type Pressure in the **Label** text field.
- 3 Locate the **Data** section. From the **Data set** list, choose **Study 2 (changing vcm)/Solution 2(sol2)**.
- 4 From the **Parameter selection (vcm)** list, choose **From list**.
- 5 From the **Parameter values (vcm)** list, choose **4, 14, 20** and **34**.
- 6 Locate the **Plot Settings** section. Select the **x-axis label** check box and type x( $\mu\text{m}$ ) in the associated text field.
- 7 Locate the **Legend** section, select the **Show legends** check box.

#### *Line Graph 1*

- 1 On the **Pressure** toolbar, click **Line Graph**.
- 2 Locate the **Selection** section. From the **Selection** list, choose **Center line**.
- 3 Locate the **y-Axis Data** section. Click **Replace Expression** in the upper-right corner of the **y-axis data** section. From the menu, choose **Model>Component 1>Creeping >Velocity and pressure>p – Pressure**.
- 4 Click to expand the **Coloring and Style** section. In the **Width** text field, type 2.
- 5 Click to expand the **Legends** section. Select the **Show legends** check box.
- 6 From the **Legends** list, choose **Manual**.
- 7 In the **Legends** table, enter the following setting:

<b>Legends</b>
vcm=4 VT

vcm=14 VT  
vcm=20 VT  
vcm=34 VT

**8** On the **Pressure** toolbar, click **Plot**.

#### *2D Plot Group 7*

- 1** On the **Home** toolbar, click **Add Plot Group** and choose **2D Plot Group**.
- 2** In the Settings window for 2D Plot Group, locate the **Data** section.
- 3** From the **Data set** list, choose **Study 2 (changing vcm)/Solution 2(sol2)**.
- 4** From the **Parameter values (vcm)** list, choose **14**.
- 5** Locate the **Legend** section, select the **Show legends** check box.

#### *Streamline 1*

- 1** Right click **2D Plot Group 7** and choose **Streamline**.
- 2** In the **Settings** window for Streamline, locate the **Streamline Positioning** section.
- 3** From the **Positioning** list, choose **Magnitude controlled**.
- 4** In the **Density** text field, type 31.
- 5** On the **2D Plot Group 7** toolbar, click **Plot**.